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Translation Symmetry Breaking in Four Dimensional Lattice Gauge Theories

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Abstract. We consider lattice gauge theories with finite abelian group G in the weak coupling regime. It is shown that there is only one translation invariant equilibrium state for the infinite system. In four dimensions we construct a nontranslation invariant equilibrium state, describing an infinite system with localized magnetic flux tube, starting and ending at infinity.

0. Introduction

It has been expected for a while (and was independently proven by Aizenman [1] and Higuchi [2]) that in two dimensions the only possible equilibrium states for the Ising model are convex combinations of the two states μ_+, μ_-, which are the thermodynamic limit of the Gibbs states in finite volume with positive respectively negative boundary conditions (b.c.). This excludes the possibility of phase coexistence and breaking of translation invariance for the two dimensional Ising system.

In three and more dimensions however such phenomena can occur at sufficiently low temperatures. This was first shown by Dobrushin in [3], who induced such a state by mixed b.c. on increasing lattices: $-$ on the lower and $+$ on the upper halves of the surface of the lattice. In the resulting infinite volume state, the average magnetization will be near $+1$ in the upper and near -1 in the lower half of the system. A considerable simplification of the proof is due to van Beijeren [4].

For lattice gauge theories, at least to my knowledge, the possibility that the corresponding states might exist and describe translation invariance breaking has not been considered in the literature. In [5], for example, it is suggested that in the region of convergence of cluster expansions the thermodynamic limit is independent of b.c. This is true in the strong coupling regime of lattice gauge theories, but for the weak coupling regime considered in this paper it is at best true for some restricted class of b.c.