Commun. Math. Phys. 96, 195–221 (1984)

Communications in Mathematical Physics

Uniqueness and the Global Markov Property for Euclidean Fields: The Case of General Exponential Interaction

Bogusław Zegarliński

Zentrum für Interdisziplinare Forschung, Universität Bielefeld, Wellinberg 1, D-4800 Bielefeld 1, Federal Republic of Germany

Abstract. The uniqueness and the global Markov property for the regular Gibbs measure corresponding to the interaction

$$U_\Lambda(\varphi) := \lambda \int_\Lambda d_2x \int d\varrho(\alpha) : e^{\alpha\varphi} :_0 (x)$$

[for $\lambda > 0$, $d\varrho(\alpha)$ a probability measure with support in $(-2\sqrt{\pi}, 2\sqrt{\pi})$] is proved.

0. Introduction

0.1. Definitions and Notations

By $\mathscr{F}$ we denote a family of bounded open sets in $\mathbb{R}^d$ partially ordered by the filtering inclusion relation $\subsetneqq$ (i.e. $\Lambda_1, \Lambda_2 \in \mathscr{F} \Rightarrow \exists \Lambda_3 \in \mathscr{F}, \Lambda_1 \subseteqq \Lambda_3, \Lambda_2 \subseteqq \Lambda_3$).

By $\mathscr{F}_0 := \{\Lambda_n \in \mathscr{F} : \Lambda_n \subseteqq \Lambda_{n+1}\}_{n \in \mathbb{N}}$ we denote a countable base of $\mathscr{F}$ (i.e. $\forall \Lambda \in \mathscr{F}, \exists \Lambda_n \in \mathscr{F}_0, \Lambda \subseteqq \Lambda_n$). We always assume that boundary $\partial\Lambda$ of $\Lambda \in \mathscr{F}$ is piecewise-C^1-curve. We assume that $\bigcup_{\mathscr{F}_0} \Lambda_n = \mathbb{R}^d$. We write $\Lambda^c := \mathbb{R}^d \backslash \Lambda$ and $\operatorname{int}\Lambda := \Lambda \backslash \partial\Lambda$.

Let $\mathscr{D} \equiv \mathscr{D}_{\text{real}}(\mathbb{R}^d)$ be the space of $C_0^\infty(\mathbb{R}^d)$ real functions and $\mathscr{S} \equiv \mathscr{S}_{\text{real}}(\mathbb{R}^d)$ the space of $C^\infty(\mathbb{R}^d)$ real rapidly decreasing functions, which are topologized as usually, e.g. [35, p. 28] for $\mathscr{D}$ and [35, p. 146] for $\mathscr{S}$. By $\mathscr{D}'$ (respectively $\mathscr{S}'$) we denote the real topological dual space of $\mathscr{D}$ (respectively $\mathscr{S}$). For $\Lambda \subseteqq \mathbb{R}^d$, we write $f \in \mathscr{D}_\Lambda$ (respectively $\mathscr{S}_\Lambda$) if $f \in \mathscr{D}$ (respectively $\mathscr{S}$) and $\sup f \subseteqq \Lambda$. Let for $f \in \mathscr{D}$ (respectively $\mathscr{S}$)

$$\varphi(f) : \mathscr{D}' \to \mathbb{R} : \mathscr{D}' \ni \eta \mapsto \varphi(f)(\eta) := \eta(f) \tag{1.1}$$

(respectively for $\mathscr{S}'$ we denote this function by the same letters). For arbitrary $\Lambda \subseteqq \mathbb{R}^d$ open Σ_Λ (respectively $\mathscr{B}_\Lambda$) is the smallest σ-algebra of subsets in $\mathscr{D}'$ (respectively $\mathscr{S}'$), such that all functions $\{\varphi(f) : f \in \mathscr{D}_\Lambda\}$ (respectively $f \in \mathscr{S}_\Lambda$) are measurable. If $\Lambda = \mathbb{R}^d$ we write $\Sigma \equiv \Sigma_{\mathbb{R}^d}$ and $\mathscr{B} \equiv \mathscr{B}_{\mathbb{R}^d}$. For arbitrary $\Lambda \subseteqq \mathbb{R}^d$ we define

$$\Sigma_\Lambda := \bigcap_{\Lambda \subset \Lambda' \text{ (open)}} \Sigma_{\Lambda'} \quad \left(\text{respectively } \mathscr{B}_\Lambda := \bigcap_{\Lambda \subset \Lambda' \text{ (open)}} \mathscr{B}_{\Lambda'}\right).$$