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On the Rate of Convergence to Equilibrium in One-Dimensional Systems*

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Abstract. We determine the essential spectral radius of the Perron-Frobenius-operator for piecewise expanding transformations considered as an operator on the space of functions of bounded variation and relate the speed of convergence to equilibrium in such one-dimensional systems to the greatest eigenvalues of generalized Perron-Frobenius-operators of the transformations (operators which yield singular invariant measures).

I. Introduction

In this note we give some estimates on the speed of convergence to equilibrium in 1-dimensional dynamical systems which can be described by a piecewise monotonic transformation $T:[0,1]\to[0,1]$. "Piecewise monotonic" means that there is a finite partition $\mathscr{I}$ of $[0,1]$ into intervals on each of which T is strictly monotone and differentiable. Throughout the paper we assume the following setting (see [4, 13]):

$g:[0,1]\to\mathbb{R}_+$ is defined by

$$g(x)=1/|T'(x)| \quad \text{for} \quad x\in X_0:=\bigcup_{I\in\mathscr{I}} \operatorname{int} I,$$

$g(x)\leqq \liminf_{y\to x, y\in X_0} g(y)$ on the finite set $[0,1]\backslash X_0$. Set

$$g_n(x)=g(T^{n-1}x)\cdot\ldots\cdot g(x)$$

and $\vartheta=\lim_{n\to\infty}(\|g_n\|_\infty)^{1/n}$, $P:L^1\to L^1$ is defined by

$$Pf(x)=\sum_{I\in\mathscr{I}}(f\cdot g)(T_{|I}^{-1}(x))\cdot 1_{TI}(x).$$

(L^1 is the space of complex-valued Lebesgue-integrable functions on $[0,1]$; P is the Perron-Frobenius-operator associated

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