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Improved Perturbation Expansion for Disordered Systems: Beating Griffiths Singularities*

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Abstract. We introduce a new expansion to prove exponential clustering of connected correlations in a large class of disordered systems. Our expansion converges for values of the temperature and magnetic field where standard cluster expansions diverge, due to the presence of Griffiths type singularities. It is organized inductively over an infinite sequence of increasing distance scales. In each induction step one redefines what is means by the "unperturbed system", a procedure somewhat reminiscent of K.A.M. theory. Our techniques may be useful in dealing with the so-called large-field problem in real-space renormalization group schemes.

1. Introduction

1.1. Overview

In this paper we introduce a new method to partially resum high-temperature, or low-activity expansions in situations where they actually diverge. Our method can be used, for example, to analyze spin glasses, the random-field Ising model and other disordered systems at temperatures and activities where straight high-temperature, or low-activity expansions diverge, due to the presence of so-called Griffiths singularities [1]. We think that our results and methods are a prerequisite for understanding critical behavior in disordered systems.

Among the mathematical problems that one encounters in the study of disordered systems are:

A. Certain random couplings, such as the spin-spin couplings, J_{ij}, in a spin glass or the inverse of the magnetic field, h_j, in a random field model, can have anomalously large values over large regions of the lattice with very small, but positive probability. In the vicinity of such regions the correlation length is

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