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Translation Group and Spectrum Condition

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Abstract. Let $\{A, \mathbb{R}^d, \alpha\}$ be a C^*-dynamical system, where $\mathbb{R}^d$ is the d-dimensional vector group. Let V be a convex cone in $\mathbb{R}^d$ and $\hat{V}$ its dual cone. We will characterize those representations of A with the properties (i) α_a, $a \in \mathbb{R}^d$ is weakly inner, (ii) the corresponding unitary representation $U(a)$ is continuous, and (iii) the spectrum of $U(a)$ is contained in $\hat{V}$.

I. Introduction

The spectrum condition is one of the essential ingredients of quantum field theory. Especially the discovery of the fact that the translations are weakly inner automorphisms for finite particle representations [4] has made the spectrum condition an interesting subject. Many problems in connection with this have been studied and answered [4–7]. In the previous investigations, which are based on the "covariance-algebra" introduced by Doplicher, Kastler, and Robinson [9], it has been assumed that the translation group is acting strongly continuous on the C^*-algebra in question. On the other hand, in the theory of local observables, one usually is only interested in representations which are locally normal with respect to the vacuum representation. But this means that the algebra associated to a bounded region should be a von Neumann algebra. Such an assumption, however, contradicts the assumption of strong continuity of the translations. Since in a recent paper [7] it has been shown that one can handle the problem of covariant representation without using the continuity of the group action on the algebra, we will treat the problem of the spectrum condition again.

Furthermore in the existing literature only the one dimensional case and its iterations have been treated with full mathematical rigour. But the case where the cone in question is an arbitrary convex cone with interior points is still missing. We also want to fill this gap.

In the next section we handle the one dimensional case again. We show that by introducing the reasonable concepts one can reduce this problem to results existing in the literature. The results obtained here are generalized in Sect. III to the n-dimensional case where the spectrum is restricted to a half space. The n-dimensional case where the spectrum is in a cone is treated in Sect. IV and V.