Commun. Math. Phys. 95, 445–486 (1984)

Communications in Mathematical Physics

Construction and Borel Summability of Planar 4-Dimensional Euclidean Field Theory

Vincent Rivasseau

Centre de Physique Théorique de l'Ecole Polytechnique, Plateau de Palaiseau, F-91128 Palaiseau, Cedex, France

Abstract. We use the methods of [1] to show that the planar part of the renormalized perturbation theory for φ_4^4-euclidean field theory is Borel-summable on the asymptotically free side of the theory. The Borel sum can therefore be taken as a rigorous definition of the $N \to \infty$ limit of a massive $N \times N$ matrix model with a $+\operatorname{tr} g\varphi^4$ interaction, hence with "wrong sign" of g. Our construction is relevant for a solution of the ultra-violet problem for planar QCD. We also propose a program for studying the structure of the "renormalons" singularities within the planar world.

I. Introduction

The standard problem in constructive field theory is to prove the existence of "models" which represent realistic interacting fields. Ultimately it should give a rigorous mathematical construction of the models which are used in the description of modern particle physics, namely the gauge theories. So far constructive field theory has not been able to provide the construction of any interacting model in 4 dimensions of space-time. To construct the 4-dimensional theories requires in our opinion a complete analysis of their relationship to renormalized perturbation theory. This does not mean that we do not believe in the existence of "non perturbative" effects in 4-dimensional gauge theories. There is good heuristic evidence for the existence of such effects. However we think that it is unlikely that a rigorous construction of these theories in the continuum can be obtained before the perturbative phenomena have been investigated in detail and brought under rigorous control.

It is known in particular that in 4-dimensional renormalizable field theories the renormalization deeply modifies the behavior of Feynman amplitudes. Some individual amplitudes become so large that alone they seem to dominate the large order behavior of perturbation theory. We think that *this* phenomenon, which has been analyzed in various heuristic ways [2–6], should be rigorously understood and controlled. A first step in this direction was accomplished in [1], where the