Asymptotic Expansion of the Logarithm of the Partition Function

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Abstract. A method is presented permitting one to find in principle all the non-decreasing terms of the asymptotic expansion of the logarithm of the partition function when the volume of region increases. The constructions are carried out at low activity for lattice systems with general n-body interactions, and continuous systems with two-body interactions.

1. Introduction

The aim of this paper is to study the asymptotic behaviour of the logarithm of a partition function $\ln \Xi(\Lambda)$ when the volume of the region Λ increases. Under natural assumptions about the grand canonical Gibbs ensemble and for appropriate classes of regions all the non-decreasing terms of the asymptotics are obtained in both cases of continuous and lattice systems.

The main term of this asymptotic (proportional to the volume $|\Lambda|$ of the region Λ) follows from the theorem by Lee and Yang [1]. For the lattice case the second term (proportional to the area of the boundary $\Gamma(\Lambda)$ of Λ] was obtained by Dobrushin under assumptions which provide the absence or the presence of a phase transition [2]. In the case of continuous systems the second term turns out to be proportional to the area of the boundary $\Gamma(\Lambda)$. The next terms depend on geometrical characteristics of $\Gamma(\Lambda)$.

The asymptotic expansion for the logarithm of the partition function is obtained in this paper as a special case of expansions for the integrals of the so-called clusterwise smooth translation-invariant functions over all finite subsets of the bounded region Λ. These asymptotic expansions are obtained for lattice systems with general n-body interactions and for continuous systems with two-body interactions.

2. Preliminaries

Consider a continuous system of particles in some bounded region Λ of the v-dimensional Euclidean space $\mathbb{R}^v$ ($v \geq 1$) interacting via pair potential $\Phi(x)$,