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The Boltzmann Equation: Global Existence for a Rare Gas in an Infinite Vacuum*

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Abstract. Solutions of the Boltzmann equation are proved to exist, globally in time, under conditions that include the case of a finite volume of gas in an infinite vacuum when the mean free path of the gas is large enough. It is also proved, as might be expected in this case, that the density of the gas at each point in space goes to zero as time goes to infinity.

1. Introduction

Here, we prove that the Boltzmann equation has a unique solution, global in time, in all space. The conditions are a) that the initial data go to zero fast enough at infinity, and b) that the mean free path is large enough. As a special case, it is illuminating to think of a finite volume of gas released into an infinite vacuum. In this paradigmatic case, our results give that the corresponding solution of the Boltzmann equation exists globally if the gas is rare enough.

The paradigm shows that infinity should be absorbing, and so it is. Under conditions a) and b), we prove in Sect. 5 that all molecules are eventually swept out of any finite domain. Thus equilibrium is always trivial. The fact that infinity is absorbing shows that our results are related to earlier work of Babovsky [1], who proved global existence in a bounded domain when the mean free path is large, assuming that the boundary of the domain is absorbing, that is, that molecules simply disappear when they reach the boundary. Our result is, perhaps, physically more interesting since particles do not literally vanish in a finite time.

To arrive at our results, we use the method of Kaniel and Shinbrot [4], which requires a global upper bound for the solution. To find this, the basic idea is that, under conditions a) and b), the dominant process in the gas should be free flow, the molecular interactions playing a secondary role. This suggests looking for an estimate

$$f(t,x,\xi) \leqq \tilde{f}(x-\xi t,\xi)\,, \tag{1.1}$$

where f is the solution of the Boltzmann equation and $\tilde{f}$, which depends on t only through the combination $x-\xi t$, describes a free flow. Estimates like (1.1) were introduced by Tartar [5] for certain discrete velocity models of the Boltzmann

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