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Bounds on Exponentials of Local Number Operators in Quantum Statistical Mechanics

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Abstract. We consider the quantum systems of interacting Bose particles confined to a bounded region Λ of the configuration spaces $\mathbb{R}^\nu$. For a class of superstable interactions we obtain bounds on exponentials of local number operators for any temperature and activity. The method we use is the Wiener integral formalism in statistical mechanics. As a consequence any thermodynamic limit states are entire analytic and locally normal in the CCR algebra. In some cases these are modular states.

I. Introduction

In this paper we study a class of superstable interactions in quantum statistical mechanics with Bose–Einstein statistics. There have been extensive studies on the thermodynamic limit in statistical mechanics of interacting quantum systems and fairly satisfactory results have been obtained for the thermodynamic functions [2, 12]. The results concerning the equilibrium states for such systems are less satisfactory. In the dilute regime, detailed properties of the thermodynamic limit states have been obtained for various classes of interactions [2–4, 6, 15]. There are also some results on the thermodynamic limits of the finite volume Gibbs states of interacting Bose particles for the charge conjugation invariant systems [5] and for the repulsive systems (with activity less than one) [1]. For the classical systems with superstable interactions, Ruelle established uniform bounds of the finite volume correlation functions [13]. Using the bounds he obtained various results on the infinite volume equilibrium states and the pressure. The results have been extended to unbounded classical spin systems [14, 8]. The main purpose of this paper is to extend Ruelle's results to quantum statistical mechanics for interacting Bose particles. The method we develop can be extended easily to unbounded quantum spin systems and will appear elsewhere [10].

We give a brief discussion of the main result. Let ρ_Λ be the Gibbs equilibrium states for a system of interacting Bose particles confined to a bounded region Λ

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