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# A Correlation Decay Theorem at High Temperature

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**Abstract.** We derive a theorem of exponential decay of correlation functions at high temperature for a general statistical mechanical system following the approach introduced by L. Gross. The theorem is formulated so as to be useful for locality problems in lattice quantum gravity.

## 1. Introduction

The purpose of the present article is to formulate a theorem of exponential decay of correlation functions at high temperature in a statistical mechanical system so as to be applicable to a lattice gravity model [1]. In fact we want to show that a lattice quantum gravity model based on Regge calculus [2] with an ultraviolet cutoff of the order of the Planck length leads to Einstein's gravity theory at large distances. The effective action derived from the lattice theory should be general coordinate invariant and local with respect to the gravitational field. The latter will be proved by the theorem of the present article in a subsequent paper [1].

There are several articles in literature discussing classical statistical mechanical systems which under certain conditions (e.g. high temperature or low activity) do not exhibit long range correlations. We refer for example to the papers of Gross [3], Israel [4], and Klein [5]. In our case we are mainly concerned with a rather generally formulated theorem introduced by Gross [3]. Gross used the Dobrushin uniqueness method [6] to derive exponential decay rates for two point correlation functions at high temperature in classical statistical mechanical lattice models. The result of Gross can be expressed in the form (Theorem 1 in [3]): If $\alpha < 1$, then for any functions $f, g \in C(\Omega)$,

$$|\sigma(fg) - \sigma(f)\,\sigma(g)| \leqq e^{-d(a,b)} \|f\|_a \|g\|_b (1-\alpha)^{-2} (1-\alpha^2)^{-1}\,. \tag{1.1}$$

$\Omega$ is the space of all functions $s$ from a countable set $L$ $(a, b \in L)$ into a compact metric space $X(s(a) \in X$ for $a \in L)$. $\sigma$ is a probability measure on $\Omega$, $d(\cdot\,,\cdot)$ is a

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