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On the Regularized Determinant for Non-Invertible Elliptic Operators

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Abstract. We propose a technique for regularizing the determinant of a non-invertible elliptic operator restricted to the complement of its nilpotent elements. We apply this approach to the study of chiral changes in the fermionic path-integral variables.

1. Introduction

In the computation of quadratic path-integrals one is naturally led to the evaluation of determinants of differential operators. These determinants clearly diverge because the eigenvalues λ_j increase without bound. Therefore, it is necessary to adopt some regularization procedure. One technique which has proved to be very useful is the ζ-function regularization [1]. Given an elliptic invertible operator D of order $m>0$, defined on a compact manifold M without boundary, of dimension n, one forms a generalized ζ-function from D by defining

$$\zeta(s,D)=\sum_j \langle D^{-s}\phi_j,\phi_j\rangle\,, \tag{1.1}$$

where $\{\phi_j\}$ is any orthonormal basis and D^{-s} is defined following Seeley [2]. For a normal D, we can take its eigenfunctions as ϕ_j's, and then (1.1) becomes

$$\zeta(s,D)=\sum_j \lambda_j^{-s}\,. \tag{1.2}$$

These series converge only for $\mathrm{Re}\,s>n/m$, but $\zeta(s,D)$ can be analytically extended to a meromorphic function of s in the whole complex plane [2]. In particular, it is regular at $s=0$.

We can define the regularized determinant of D, $\mathrm{Det}(D)$, as

$$\mathrm{Det}(D)=\exp\left(-\frac{d\zeta}{ds}(s,D)\right)\Bigg|_{s=0}. \tag{1.3}$$

Note that, for a normal D, since the ζ-function is given by Eq. (1.2), its derivative at $s=0$ is formally equal to $-\sum_j \ln\lambda_j$, and then Eq. (1.3) turns to be the regularization of the product of the eigenvalues of D.