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The Twistor Connection and Gauge Invariance Principle

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Abstract. It is shown that the twistor connection of the local twistor theory can be regarded as a gauge field whose Yang-Mills equations are equivalent to Bach equations of gravity.

1. Introduction

It is well known that the affine connection of Einstein's theory of gravity can be regarded as a gauge field. But consistent interpretation of the gravitation as a gauge field leads, strictly speaking, to non-Einstein theory, since the affine connection itself (not a metric tensor) turns out to be the basic field variable and the most natural gravitational Lagrangian should be quadratic in Riemann's curvature tensor [4]. Anyhow, two groups of field equations can be obtained in this approach:

(1) variations of the gravitational Lagrangian for the connection lead to (so-called) quasi-Maxwell equations for Riemann's curvature tensor (these are Yang-Mills equations);

(2) variations of the gravitational Lagrangian for the metric tensor lead to (a generalization of) Einstein's equations (gravity field equations).

It should be stressed that in the case of the affine connection Euler-Lagrange equations of these variational principles (with *any* Lagrangian) are *different*, i.e. Yang-Mills equations do not coincide with gravity field equations.

In this work it is shown that the local twistor covariant derivative may be regarded as a gauge covariant derivative, resulting from the localization of the subgroup of the twistor group SU(2, 2), which describes transformations of the local twistor components under conformal rescalings. In Sect. 2 the standard gauge transformation law of the Yang-Mills formalism is slightly modified to treat such cases, and in Sect. 4 it is proved that the twistor connection is a gauge field. In Sect. 5 a twistor connection Lagrangian is constructed and it is shown that the variational principle (1) (which gives sixty equations, fifty of which are zero identically) and the variational principle (2) (which gives ten equations) lead to the *same* conformally-invariant equations of gravity-vanishing of the Bach tensor.

The following index conventions are used: lower case Latin indices will be used for tensors, upper case Latin indices for spinors, and Greek indices for twistors. The sign convention for the curvature adopted here is that $[\nabla_c, \nabla_d] V_b = R^{a}_{\cdot bcd} V_a$, and $R^{c}_{\cdot acb} = R_{ab}$ [2].