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Lattice Dipole Gas and $(\nabla\phi)^4$ Models at Long Distances: Decay of Correlations and Scaling Limit

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Abstract. We prove that the scaling limit for a large class of weak $V(\nabla\phi)$ perturbations of the free massless lattice field ϕ is Gaussian with the covariance $c(V)(-\Delta)^{-1}$. The correlations as well as $c(V)$ are analytic in V. In particular the Mayer series for the dipole gas is convergent for small activity.

1. Introduction

The authors have been pursuing a program to gain a rigorous control of asymptotically free (AF) models of statistical mechanics and quantum field theory. This paper finishes such an analysis for infrared (IR) AF models, such as the dipole gas, $(\nabla\phi)^4$ model and related ones. We show that their correlations become those of a free massless field at long distances: the canonical scaling limit is shown to be the massless Gaussian Euclidean field with a definite field strength renormalization.

In a previous paper [1] the authors studied the renormalization group (RG) trajectory of the Hamiltonian in a general space of Hamiltonians. This analysis is now applied to the study of the correlations. The results of the present paper may also be interpreted as setting up rigorously the RG in a space of Gibbs states of certain critical (massless) theories and showing the convergence of its iterations to the state given by the massless Gaussian fixed point, in the sense of convergence of correlations. We, however, state our results only pragmatically, as a result about scaling limits and IR properties of the correlations.

When [1] was finished we obtained [2] where infrared behavior of the weakly coupled $(\nabla\phi)^4$ model was controlled by means of a phase-cell expansion. Both methods are similar as they are based on an analysis of contributions of different momenta on different scales of distances. In [2] different momentum scales are entangled in the expansion whereas we analyze the contribution of one momen-

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