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Exponential Bounds and Semi-Finiteness of Point Spectrum for N-Body Schrödinger Operators

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Abstract. For a large class of N-body Schrödinger operators H, we prove that eigenvalues of H cannot accumulate from above at any threshold of H. Our proof relies on L^2 exponential upper bounds for eigenfunctions of H with explicit constants obtained by modifying methods of Froese and Herbst.

In this note we study the point spectrum of certain N-body Schrödinger operators. To specify them, let $m_i > 0$ and $x_i \in \mathbb{R}^\nu$, $1 \leqq i \leqq N$, denote the mass and position of the i^{th} particle, let $x \in \mathbb{R}^{N\nu}$ be given by $x = (x_1, \ldots, x_n)$, and let

$$X = \left\{ x \in \mathbb{R}^{N\nu} : \sum_{i=1}^{N} m_i x_i = 0 \right\}$$

with norm

$$|x|^2 = \sum_{i=1}^{N} 2 m_i x_i \cdot x_i ,$$

where $\cdot$ is the usual inner product on $\mathbb{R}^\nu$. We consider operators H on $L^2(X, dv)$ (with volume measure determined by the norm $|\cdot|$) of the form

$$H = -\Delta_X + \sum_{1 \leqq i < j \leqq N} V_{ij}(x_i - x_j) ,$$

where $-\Delta_X$ is the Laplace-Beltrami operator on X and the $V_{ij}(y)$ are real-valued, measurable functions on $\mathbb{R}^\nu$. Throughout, we assume that

$$V_{ij}(-\Delta + 1)^{-1} \quad \text{and} \quad (-\Delta + 1)^{-1}(y \cdot \nabla V_{ij})(-\Delta + 1)^{-1}$$
$$\text{are compact as operators on } L^2(\mathbb{R}^\nu, d^\nu y),\ 1 \leqq i, j \leqq N. \tag{1}$$

Here $-\Delta$ is the Laplacian on $L^2(\mathbb{R}^\nu, d^\nu y)$ and ∇V_{ij} is the distributional gradient of V_{ij}. Under these assumptions, H is well-defined as an operator perturbation of $-\Delta_X$ and the crucial "Mourre estimate" holds [1, 4, 6].

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