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Convergence of Grand Canonical Gibbs Measures*

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Abstract. We prove existence of Gibbs states for a large class of continuum many-body potentials through a limit process of hard-core approximations to the potential. Dobrushin uniqueness techniques [1] for the decay of correlations are then extended to a very general class of continuum potentials.

Introduction

Ruelle [11] proved existence of grand canonical Gibbs states for a large class of pair potentials in continuum statistical mechanics. In this paper we prove, by a different method, the existence of Gibbs states for a large class of continuum many-body potentials. The approach we take is to add a "hard-core N-body component" φ_N to a given potential V. φ_N has the effect of restricting the number of particles that can accumulate in a spherical region of space of diameter r_0 to no more than N. Existence of a Gibbs state σ_N for the potential $V+\varphi_N$ is easily established for each positive integer N. Using some standard theorems on convergence of probability measures, we prove that the Gibbs states $\{\sigma_N\}$ converge to a Gibbs state σ for the potential V.

We then apply these methods to extend results on the decay of averaged two point correlation functions established by Gross [4], Künsch [6], Föllmer [3], and the author [5] via Dobrushin uniqueness techniques [1]. In addition we show that the grand canonical pressure P_N for $V+\varphi_N$ converges to the pressure P for V as N approaches infinity.

Section 1. Notation and Definitions

For a Borel measurable subset $\Lambda \subset \mathbb{R}^d$, let $X(\Lambda)$ denote the set of all locally finite subsets of Λ. $X(\Lambda)$ represents configurations of identical particles in Λ. We let $\emptyset$ denote the empty configuration. Let B_Λ be the σ-field on $X(\Lambda)$ generated by all sets of the form $\{s \in X(\Lambda): |s \cap B| = m\}$, where B runs over all bounded Borel subsets of Λ, m runs over the set of nonnegative integers, and

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