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Leading Large Order Asymptotics for $(\phi^4)_2$ Perturbation Theory*

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Abstract. We develop a rigorous semiclassical expansion to compute the radius of convergence of the Borel transform for the pressure in $(\phi^4)_2$ field theory. This result gives a partial justification for the Lipatov method of finding large order perturbation theory asymptotics in quantum field theory.

1. Introduction

The Lipatov method is an interesting formal technique for finding the large order behavior of the perturbation coefficients in certain divergent perturbation series. The real power of this method is that it is applicable to the perturbation series which occur in quantum field theory. The basic idea of the approach, initiated by Lipatov [1] and extensively developed by Brezin et al. [2–4], is to use a path integral representation for the k^{th} perturbation coefficient in the perturbation series and then to do a formal semiclassical expansion of the path integral as $k \to \infty$. Knowledge of the large order behavior of perturbation theory may be combined with summability methods, such as Padé or Borel summation, to do numerical calculations (see [5] and other articles in the same volume for more details). Of particular interest for field theory are the calculations of critical exponents done by Le Guillou and Zinn-Justin [6–8] based on the perturbation theory asymptotics of [2]. Our result described in the abstract gives a partial justification, for $(\phi^4)_2$ field theory, of these large order asymptotics and of the Lipatov method of deriving them.

In order to state our result and describe its connection with the Lipatov method calculations, consider a $(\phi^4)_2$ field theory with partition function

$$Z_X(\lambda) = \int e^{-\lambda V(\phi)} d\mu_\Lambda^X$$

in which $V(\phi) = \int_\Lambda :\phi^4(x): d^2x, \Lambda = [-T/2, T/2]^2$, and $X = p$ (periodic), D

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