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On a Sharp Transition from Area Law to Perimeter Law in a System of Random Surfaces*

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Abstract. We introduce and study a phase transition which is associated with the spontaneous formation of infinite surface sheets in a Bernoulli system of random plaquettes. The transition is manifested by a change in the asymptotic behavior of the probability of the formation of a surface, spanning a prescribed loop. As such, this transition offers a generalization of the bond percolation phenomenon. At low plaquette densities, the probability for large loops is shown to decay exponentially with the loops' area, whereas for high densities the decay is by a perimeter law. Furthermore, we show that the two phases of the three dimensional plaquette system are in a precise correspondence with the two phases of the dual system of random bonds. Thus, if a natural conjecture about the phase structure of the bond percolation model is true, then there is a sharp transition in the asymptotic behavior of the surface events. Our analysis incorporates block variables, in terms of which a non-critical system is transformed into one which is close to a trivial, high or low density, fixed point. Stochastic geometric effects like those discussed here play an important role in lattice gauge theories.

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