Commun. Math. Phys. 9, 1–18 (1983)

Communications in Mathematical Physics

Surviving Extrema for the Action on the Twisted SU(∞) One Point Lattice

Pierre van Baal

Institute for Theoretical Physics, Princetonplein 5, P.O. Box 80.006, NL-3508 TA Utrecht, The Netherlands

Abstract. We give a simplified construction of "twist eating" configurations, based on a theorem due to Frobenius. These configurations are defined through the equation: $U_\mu U_\nu U_\mu^+ U_\nu^+ = \exp(2\pi i n_{\mu\nu}/N)$ with $U_\mu \in \mathrm{SU}(N)$, $\mu = 1$ to d and $n_{\mu\nu}$ an antisymmetric matrix with integer entries. In the (Twisted)-Eguchi-Kawai model they yield extrema some of which survive for $N \to \infty$. Comparison is made with the Monte Carlo data of the internal energy in the small coupling region.

1. Introduction

The recently introduced Twisted-Eguchi-Kawai (TEK) model [2] combines four powerful approaches to the strong interaction theories. The $1/N$-expansion [3], the lattice approximation to space time [4], the twisted boundary condition for gauge fields in a box [5] and the loop-equations [6]. For the Eguchi-Kawai model [1] to work, one needs zero vacuum expectation values for all open loops. This is only guaranteed in the strong coupling region. Quenching [7] has been introduced to extend it to all coupling. It however considerably complicates calculations, unlike the twisted version, where the model is defined by a simple modification of the action:

$$S = \sum_{\mu \neq \nu = 1}^{4} \mathrm{Tr}(1 - Z_{\mu\nu} U_\mu U_\nu U_\mu^+ U_\nu^+) . \tag{1}$$

Here U_μ are the $\mathrm{SU}(N)$ link variables on a one point lattice and $Z_{\mu\nu} = Z_{\nu\mu}^*$, an element of the centre Z_N of $\mathrm{SU}(N)$, is the twist. It is labelled by the twist tensor $n_{\mu\nu}$ through:

$$Z_{\nu\mu}(n) = \exp(2\pi i n_{\mu\nu}/N) , \tag{2}$$

where $n_{\mu\nu}$ is an antisymmetric 4×4 matrix with integer entries defined modulo N.

The lower bound of the action (1) is zero and is saturated if and only if there exist four elements Ω_μ of $\mathrm{SU}(N)$ satisfying:

$$\Omega_\mu \Omega_\nu \Omega_\mu^+ \Omega_\nu^+ = Z_{\nu\mu}(n) . \tag{3}$$