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Euler Evolution for Singular Initial Data and Vortex Theory

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Abstract. We study the evolution of a two dimensional, incompressible, ideal fluid in a case in which the vorticity is concentrated in small, disjoint regions of the physical space. We prove, for short times, a connection between this evolution and the vortex model.

1. Introduction

In this paper we want to study some properties of the behaviour of a non-viscous, incompressible fluid in two dimensions. The Euler equations for the vorticity of such a fluid in all $\mathbb{R}^2$ are:

$$\left.\begin{aligned} &\frac{\partial}{\partial t}\omega(x,t)+(u\cdot\nabla)\omega(x,t)=0\,, \qquad \nabla\cdot u=0\,,\\ &\omega(x,t)=\operatorname{curl} u(x,t)=\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right)(x,t)\,, \qquad x=(x_1,x_2)\in\mathbb{R}^2\,,\\ &\omega(x,0)=\omega_0(x)\,. \end{aligned}\right\} \tag{1.1}$$

Here $u=(u_1,u_2)\in\mathbb{R}^2$ denotes the velocity field.

If u decays at infinity, the incompressibility condition allows us to reconstruct the velocity field by means of ω. In fact, by $\nabla\cdot u=0$, there exists a function Ψ, such that $u=\nabla^\perp\Psi$, where $\nabla^\perp=\left(\frac{\partial}{\partial x_2}, -\frac{\partial}{\partial x_1}\right)$. Hence $\Delta\Psi=-\omega$ and

$$u(x,t)=\int k(x-y)\omega(y)dy\,, \qquad k=\nabla^\perp g\,, \tag{1.2}$$

$$g(r)=-\frac{1}{2\pi}\ln|r| \qquad r\in\mathbb{R}^2\,. \tag{1.3}$$

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