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Generic Fréchet Differentiability of the Pressure in Certain Lattice Systems

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Abstract. This note has two goals. The first is to give an explicit description (Theorem 1) of the duals of certain weighted products $\mathscr{B}_h$ of a countable family of Banach spaces. These products include the usual spaces of interactions which arise in statistical mechanics. The second goal is to use this description to prove that if the factor spaces are finite dimensional and the weight function h satisfies a certain growth condition, then the pressure is Fréchet differentiable wherever it is Gateaux differentiable (hence is Fréchet differentiable in a dense G_δ subset).

1. Notation and Definitions

The set-up described below is a slight generalization of a standard one for lattice systems. Let $\mathbb{Z}^\nu$ denote the ν-dimensional integer lattice. Throughout the paper the letters X and Y will denote non-empty finite subsets of $\mathbb{Z}^\nu$. We assume that to each X there corresponds a real Banach space $\mathfrak{A}_X$ containing an element 1_X of norm 1. For each ν-tuple i in $\mathbb{Z}^\nu$ we let τ_i be an isometry from $\mathfrak{A}_X$ onto $\mathfrak{A}_{X+i}$, with $\tau_i 1_X = 1_{X+i}$ and $\tau_i\tau_j = \tau_{i+j}$.

Definition. Let h be a positive function on the non-empty finite subsets X of $\mathbb{Z}^\nu$ satisfying $h(X+i) = h(X)$ for each such X and each i in $\mathbb{Z}^\nu$. Let $\mathscr{B}_h$ denote the Banach space of all functions Φ on the non-empty finite subsets X of $\mathbb{Z}^\nu$ which satisfy

$$\Phi(X) \in \mathfrak{A}_X \qquad \text{(for each such } X\text{)},$$
$$\tau_i\Phi(X) = \Phi(X+i) \qquad \text{(each } X\text{, each } i \text{ in } \mathbb{Z}^\nu\text{)},$$

and

$$\sum_{0\in X} h(X)\|\Phi(X)\| < +\infty\,,$$

with norm

$$\|\Phi\| = \sum_{0\in X} h(X)\|\Phi(X)\|\,.$$