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The Instability of Fronts in a Porous Medium*

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Abstract. We use a random choice numerical method to analyze the instability of a front separating two fluids in a porous medium. We observe a linear instability and a catastrophic finite amplitude instability. A qualitative analogy with problems involving a transition to turbulence is pointed out.

Introduction

The goal of this paper is to present an analysis of the onset of instability in a front separating two fluids in a porous medium. The analysis will be based on Glimm's (random choice) numerical method. The front is known to be unstable for sufficiently large values of a parameter μ, the viscosity ratio. We shall show that there exist two kinds of instability: for $\mu \geqq \mu_0 = 3$, small perturbations that are spatially smooth will grow slowly: for $\mu \geqq \mu_1 < \mu_0$, perturbations of large enough amplitude grow catastrophically. The two types of instability can interact.

The results and the numerical method are of practical significance in problems of oil flow and resevoir engineering. A random choice method [13, 3, 4] has been previously applied to such problems by Concus et al. [1, 10], Glimm et al. [14, 15], and Lotstedt [19]. Our method differs from earlier work in several respects, the most important of which is the fact that we keep some two dimensional information in order to reduce the possibility that one dimensional sweeps misinterpret the nature of waves moving diagonally across the grid. The possibility of large errors in this situation has been pointed out by Crandall and Majda [11, 12] and by Colella [9]. We do not track fronts.

The results regarding the different kinds of instability resemble strongly other phenomena previously observed in hydrodynamics (see e.g. [5, 7, 22]). We have a

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