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Unitary Representations of Non-Compact Supergroups

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Abstract. We give a general theory for the construction of oscillator-like unitary irreducible representations (UIRs) of non-compact supergroups in a super Fock space. This construction applies to all non-compact supergroups G whose coset space G/K with respect to their maximal compact subsupergroup K is "Hermitean supersymmetric". We illustrate our method with the example of SU$(m, p/n+q)$ by giving its oscillator-like UIRs in a "particle state" basis as well as "supercoherent state basis". The same class of UIRs can also be realized over the "super Hilbert spaces" of holomorphic functions of a Z variable labelling the coherent states.

1. Introduction

Lie groups play a fundamental role in the formulation of modern physical theories. The continuous symmetries observed in nature find expression in terms of them. The theory of their unitary representations is a well-established chapter of mathematics in the compact case. The rotation group and essentially all internal symmetry groups such as isospin $SU(2)_I$ or colour $SU(3)_C$ are compact. On the other hand, most space-time symmetry groups such as the Lorentz group, Poincaré group and conformal group are non-compact. The general theory of the unitary irreducible representations (UIRs) of non-compact Lie groups is however not yet at the same stage of completion as the compact case [1].

About a decade ago a new kind of symmetry principle entered physics, namely supersymmetry [2]. The novel feature of this symmetry is that it operates between bosons and fermions which have different space-time (or spin and statistics) properties. The generators of supersymmetry transformations form a Lie superalgebra whose *even* subalgebra is an ordinary Lie algebra. The *odd* generators corresponding to transformations between bosons and fermions close into the even subalgebra under anti-commutation. A complete classification of the simple

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