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Convergence of the Viscosity Method for Isentropic Gas Dynamics

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Abstract. A convergence theorem for the method of artificial viscosity applied to the isentropic equations of gas dynamics is established. Convergence of a subsequence in the strong topology is proved without uniform estimates on the derivatives using the theory of compensated compactness and an analysis of progressing entropy waves.

1. Introduction

We are concerned with the zero diffusion limit for hyperbolic systems of conservation laws. The general setting is provided by a system of n equations in one space dimension,

$$u_t + f(u)_x = 0, \tag{1.1}$$

where $u = u(x,t) \in R^n$ and f is a smooth nonlinear map defined on a region Ω of R^n. The zero diffusion limit is concerned with the convergence of approximate solutions to (1.1) generated by parabolic regularization. In this paper we shall deal with the Cauchy problem for diffusion processes of the classical form

$$u_t + f(u)_x = \varepsilon D(u)_{xx}, \tag{1.2}$$

and we shall establish, in particular, a convergence theorem for the method of artificial viscosity applied to the isentropic equations of gas dynamics with a polytropic equation of state

$$\begin{aligned} &\varrho_t + (\varrho u)_x = 0, \\ &(\varrho u)_t + (\varrho u^2 + p)_x = 0, \qquad p = \text{const}\, \varrho^\gamma. \end{aligned} \tag{1.3}$$

The conservation laws of mass and momentum (1.3) may, of course, be formulated in terms of the primitive densities ϱ and $m = \varrho u$ to yield the form (1.1):

$$\begin{aligned} &\varrho_t + m_x = 0, \\ &m_t + (m^2/\varrho + p)_x = 0. \end{aligned}$$