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Difficulties with Massless Particles?

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Abstract. Some difficulties with sharp momentum (one-particle) states for massless particles are indicated, in the framework of unitary irreducible representations of the Poincaré group. It is shown that a Poincaré covariant set of such states requires the introduction, in the spatial direction opposite to the point stabilized, of momentum generalized eigenstates which (when the helicity is nonzero) have a nontrivial orbital transformation. The relevance of these generalized momentum eigenstates for massless theories is then shown

1. Introduction

The basis of all the existing modern fundamental particle theories is a family of interacting massless fields. This is so in gravity, in the electroweak model before spontaneous symmetry breaking, and in quantum chromodynamics (at least in the original version). For this reason massless particles became even more important in the last decade or so

In spite of a few recurring claims that from some point of view the massless limit can be considered smooth [1], singularities do appear in the massless limit. Even the kinematics of one-particle massless particles presents a picture that is entirely different from massive particle kinematics.

In this paper we shall show that the concept of sharp momentum states, used elsewhere in physics with relative safety, has to be treated with great care as far as massless particles are concerned. In fact, distributions of the δ-function type are not relevant everywhere—sometimes they must be replaced by "twisted δ-functions" that, though supported on a single point, carry angular momentum.

A Paradox

Consider a massless state with momentum $\mathbf{p}$ and helicity $j \neq 0$, commonly defined by

$$\frac{1}{|\mathbf{p}|}\,\mathbf{p}\cdot\mathbf{J}|p,j\rangle = j|p,j\rangle \tag{1}$$

If $\mathbf{p}$ points along the positive (negative) third coordinate axis, then the third