

The Positivity of Gravitational Energy and Global Supersymmetry

C. M. Hull

Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge, CB3 9EW, England

Abstract. The concept of gravitational energy and the proof of its positivity are reviewed. The relationship between Witten's proof of the positivity of mass and supergravity is explained with reference to the group of global supersymmetries of a spacetime. A formula for the mass is given, in terms of the change of the supercharge under global supersymmetry, which has a simple positivity theorem and which reduces to Witten's expression. An interpretation of Witten's constraint on the spinors used in his proof is given.

Introduction

In special relativistic theories there is a conserved total 4-momentum and angular momentum associated with the translational and rotational invariance of flat space [1]. These first integrals of motion give important constraints on the dynamics, independent of the details of the mechanisms involved.

In general relativity, on the other hand, space–time is curved and in general has no global group of isometries. If the space–time has suitable asymptotic behaviour, however, one can define conserved charges associated with the generators of the group of motions of the asymptotic region. The case most often considered is that of a space whose metric tends asymptotically to that of Minkowski space, de Sitter space or anti-de Sitter space with the Poincaré group or the (anti-) de Sitter group as the asymptotic group. In particular, the energy is the charge associated with time translations.

Gravitational potential energy is negative since gravity is attractive. In Newtonian theory, the potential is unbounded below so that the energy of a gravitating system can be made arbitrarily negative by reducing its size, while in Einstein's theory the potential is even more negative. The energy of the gravitational field is non-local, since the Equivalence Principle implies that the strictly local effects of gravity can be removed by choosing a local inertial frame. The total energy receives positive contributions from matter and negative contributions from gravitational potential energy and so could, in principle, be of either sign.