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The Conserved Currents for the Maxwellian Field

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Abstract. We classify the conserved currents for the Maxwellian field. There are four families: (1) the classical currents derived using Noether's theorem from conformal invariance (2) certain Noetherian currents based on translations in field space, (3,4) two more kinds not equivalent to any Noetherian form.

1. Introduction

E. Noether first discovered that symmetries for a field theory gave rise to conserved currents (see [2]). The two concepts are not coextensive. We show below that certain well-known conserved currents (the rows of the symmetrized energy-momentum tensor) are not the result, so to speak, of any symmetries of the Maxwellian (electromagnetic) field.

This paper consists of two parts. (1) We enumerate the conserved currents: they fall into four classes. Two of them are the Noetherian forms already known, being those due to the conformal invariance, and the translational invariance in field-space (i.e. the transformation of the general field by adding a specific field). The third kind includes the parts added to the current of the first kind when one symmetrizes the energy-momentum tensor. The fourth kind cannot be described in a few words. (2) We prove that the third and fourth kind can never be equivalent to any Noetherian form except in degenerate cases.

As a corollary we obtain a description of all symmetries of the Maxwell system: They are just those already mentioned.

2. Main Theorem on Dynamic Currents

We take $\mathbb{R}^4$ as our model for space–time M and use t^1, t^2, t^3, t^4 for the Cartesian coordinates there. We use u_1, u_2, u_3, u_4 as the Cartesian Coordinates in the space $Q(=\mathbb{R}^4)$, where the field (the 4-vector potential) has its values.

A first order *jet* from M to Q is a linear map j from some tangent vector space $T^1(M,a)$ of M to some $T^1(Q,b)$ of Q. A function U from M to Q defines a jet j at each a in M with $b = U(a)$. It has coordinates $t^i(j) = t^i(a)$, $u_i(j) = u_i(U(a))$. It also has 16