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An Uncertainty Principle for Fermions with Generalized Kinetic Energy

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Abstract. We derive semiclassical upper bounds for the number of bound states and the sum of negative eigenvalues of the one-particle Hamiltonians $h = f(-i\nabla) + V(x)$, acting on $L^2(\mathbb{R}^n)$. These bounds are then used to derive a lower bound on the kinetic energy $\sum_{j=1}^{N} \langle \psi, f(-i\nabla_j)\psi \rangle$ for an N-fermion wavefunction ψ. We discuss two examples in more detail: $f(p) = |p|$ and $f(p) = (p^2 + m^2)^{1/2} - m$, both in three dimensions.

1. Introduction

In this paper we present upper bounds for the number of bound states $N(V)$, and the absolute value $S(V)$ of the sum of negative eigenvalues for the single particle Hamiltonians $f(-i\nabla) + V(x)$, acting on $L^2(\mathbb{R}^n)$. These bounds are then used to derive a lower bound for the kinetic energy, associated with $f(-i\nabla)$, of a system of N fermions.

In the case where $f(p) = p^2$, these bounds are well-known. One has (see e.g. [1], XIII.3 for a review)

$$N(V) \leqq C_n \int d^n x |V(x)|^{n/2}, \qquad (n \leqq 3) \tag{1.1}$$

$$S(V) \leqq C'_n \int d^n x |V(x)|^{1+n/2}, \qquad (n \leqq 1). \tag{1.2}$$

A bound of type (1.2) was first obtained in [2]; later several different and independent proofs for (1.1) were given [3–5]. The best value for the constant C_3 was obtained in [4].

Using a technique given in [2, 6], one can derive from (1.2) the following lower bound on the kinetic energy of an N-fermion-system:

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