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Bifurcation to Infinitely Many Sinks

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Abstract. This paper considers one parameter families of diffeomorphisms $\{F_t\}$ in two dimensions which have a curve of dissipative saddle periodic points P_t, i.e. $F_t^n(P_t)=P_t$ and $|\det DF_t^n(P_t)|<1$. The family is also assumed to create new homoclinic intersections of the stable and unstable manifolds of P_t as the parameter varies through t_0. Gavrilov and Silnikov proved that if the new homoclinic intersections are created nondegenerately at t_0, then there is an infinite cascade of periodic sinks, i.e. there are parameter values t_n accumulating at t_0 for which there is a sink of period n [GS2, Sect. 4]. We show that this result is true for real analytic diffeomorphisms even if the homoclinic intersection is created degenerately. We give computer evidence to show that this latter result is probably applicable to the Hénon map for A near 1.392 and B equal -0.3.

Newhouse proved a related result which showed the existence of infinitely many periodic sinks for a single diffeomorphism which is a perturbation of a diffeomorphism with a nondegenerate homoclinic tangency. We give the main geometric ideas of the proof of this theorem. We also give a variation of a key lemma to show that the result is true for a fixed one parameter family which creates a nondegenerate tangency. Thus under the nondegeneracy assumption, not only is there a cascade of sinks proved by Gavrilov and Silnikov, but also a single parameter value t^* with infinitely many sinks.

1. Introduction

The existence of a cascade of sinks is important because it analyzes a sequence of bifurcation which is different than period doubling. The existence of infinitely many sinks in Theorem C shows that there are generic situations which often arise where points tend to infinitely many distinct attractors. It indicates that for certain parameter values near $A=1.392$ the Hénon map does not have a transitive strange attractor but actually many different periodic sinks with narrow basins of attraction. (See Example 2.4 below.)

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