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Does a Generic Connection Depend Continuously on its Curvature?

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Abstract. For a principal bundle with semi-simple structure group over a smooth four-dimensional base manifold, the set of connections (gauge potentials) A which are uniquely determined by their curvature (field or field strength) F is generic in the set of all potentials, endowed with the Whitney C^∞ topology. However, the operator taking each such field F to its potential A is not continuous. Partial negative results are given concerning the existence of a smaller generic set on which this operator is continuous.

0. Introduction

Wu and Yang [WY] showed that one new aspect of non-abelian gauge theories is the existence of field copies, i.e., gauge inequivalent potentials (connections) with the same gauge field (=field strength=curvature). Since then there has been an interest in describing, either completely or modulo gauge equivalence, the set of gauge potentials with a given gauge field. (See, for example, [C, DX, DD, DW, GY, H1, H2, H3, KC, M, R, So, W].)

The fact that for many fields the potential is unique leads one to another set of questions. What characterizes the set $\mathscr{A}^*$ of gauge potentials which are uniquely determined by their curvature? Is $\mathscr{A}^*$ generic in the space $\mathscr{A}$ of all connections? What can be said about the mapping $\mathscr{F}$ of function spaces (endowed with the respective Whitney C^∞ topologies) taking each potential A to its field F? Is the inverse of the restriction $\mathscr{F}|\mathscr{A}^*$ continuous? That is, can one derive estimates for these potentials and their derivatives from similar estimates on the field?

These questions are of interest because they relate to the possibility of doing quantum field theory in the Feynman approach using functional integrals over the space of gauge fields, where the gauge transformations φ act tensorially ($F\mapsto\varphi^{-1}F\varphi$) as opposed to the space of potentials, where the action is affine and depends on the derivative of the gauge transformation ($A\mapsto\varphi^{-1}A\varphi+\varphi^{-1}d\varphi$) [H2]. Other advantages of working on the space of gauge fields instead of gauge potentials are listed in another paper by Halpern [H3].

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