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Surface Models with Nonlocal Potentials: Upper Bounds

C. Eugene Wayne*

Institute for Mathematics and its Applications, University of Minnesota, Vincent Hall, 206 Church Street S.E., Minneapolis, MN 55455, USA

Abstract. The behavior of fluctuations in a class of surface models with exponentially decaying nonlocal potentials in studied. Combining a Mayer expansion with a duality transformation, we demonstrate the equivalence of these models to a class of two dimensional spin systems with nonlocal interactions. The expansions give sufficient control over the potentials to allow the fluctuations to be bounded from above by the means of complex translations in the spin representation of the model.

1. Introduction

In this paper a class of models obtained by introducing nonlocal potentials into the solid-on-solid (SOS) model is studied. We show that for a certain class of potentials there exists a finite positive constant $c(\beta, J)$ such that fluctuations in the interface described by the model may be bounded by

$$\langle (h_0 - h_x)^2 \rangle \leqq c(\beta, J) \ln(1 + |x|), \tag{1.1}$$

for all non-zero inverse temperatures β.

The models considered have finite volume partition functions

$$\mathbf{Z}_\Lambda = \sum_{\{h\}} \exp\Big(-\beta \sum_{\langle i,j \rangle} |h_i - h_j| + \sum_{X \subset \Lambda} V_X^J(\{h\}|_X) \Big). \tag{1.2}$$

Let Λ be a square region in $\mathbb{R}^2$, centered at the origin, of side length $(2m+1)$, $m \in \mathbb{Z}$. The sum over $\{h\}$ runs over all configurations of integer valued fields on $\mathbb{Z}^2$, which obey the boundary conditions $h_i \equiv 0$, for all sites $i \in \mathbb{Z}^2 \cap \Lambda^c$. For technical reasons we require that $|\Lambda| > \Lambda_0$, where $|\Lambda|$ is the number of sites in $\mathbb{Z}^2 \cap \Lambda$, and Λ_0 is some constant defined in the appendix. Throughout this paper, $\langle i, j \rangle$ will denote a pair of nearest neighbor sites in $\mathbb{Z}^2$. Because of our boundary conditions, the sum over nearest neighbor pairs may be thought of as running over all pairs in $\mathbb{Z}^2$, or only over those pairs which intersect Λ. The sum over $X \subset \Lambda$ runs over all connected sets

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