Commun. Math. Phys. 90, 251–262 (1983)

Communications in Mathematical Physics

A Markov Dilation of a Non-Quasifree Bloch Evolution

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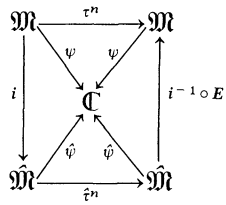
Abstract. We construct a new minimal dilation of a dynamical system governed by a Bloch equation. In contrast to a dilation of the same dynamical system recently obtained by Varilly [13] our dilation satisfies a Markov property. This presents the first example of a Markov dilation for a non-commutative dynamical system which is not equivalent to a quasifree evolution. Furthermore the dilation turns out to be a generalized K-system.

1. Introduction

In the operator algebraic framework of quantum statistical mechanics we define a *(continuous) dynamical system* by a triple $(\mathfrak{M}, \psi, \tau)$, where $\mathfrak{M}$ is a W^*-algebra, ψ is a faithful normal state on $\mathfrak{M}$, and τ is a (pointwise weak* continuous) representation of the semigroup $\mathbb{N}$ (respectively $\mathbb{R}_+$) as completely positive identity preserving operators on $\mathfrak{M}$ leaving ψ invariant. In particular, if these operators are *-automorphisms, $(\mathfrak{M}, \psi, \tau)$ will be called a *conservative dynamical system.*

Given a dynamical system $(\mathfrak{M}, \psi, \tau)$, various reasons raise the problem of constructing a conservative dynamical system $(\hat{\mathfrak{M}}, \hat{\psi}, \hat{\tau})$ containing it in the following sense. There is an injective *-representation $i : \mathfrak{M} \to \hat{\mathfrak{M}}$ and a projection E of norm one of $\hat{\mathfrak{M}}$ onto $i(\mathfrak{M})$ such that the diagram



* Research supported in part by Studienstiftung des deutschen Volkes