Commun. Math. Phys. 90, 175–186 (1983)

Communications in Mathematical Physics

Hedgehogs in a Three-Dimensional Anisotropic Spin System

Thordur Jonsson
Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Abstract. We study a continuum version of a classical anisotropic spin model in three dimensions with three component spins. We prove the existence of topological defects, called hedgehogs, which are analogous to the vortices in the two-dimensional xy-model and have a logarithmically divergent action. Bounds for the interaction energy of a hedgehog and an antihedgehog are derived.

1. Introduction

It has been known for some time now that topological defects can play an important role in the statistical mechanics of materials as well as in field theories. In this paper we study defects that arise in a classical continuum spin system (nonlinear σ-model) whose energy (action) is given by the anisotropic functional

$$A(\mathbf{S}) = \int \{(\partial_z \mathbf{S})^2 + (\Delta \mathbf{S})^2\} d^3x. \tag{1.1}$$

Here $\mathbf{S}$ is a three component spin vector of unit length, x, y, z are Cartesian coordinates in $\mathbb{R}^3$ and Δ is the Laplacian in the xy-plane.

The action (1.1) comes up in the study of the Lifshitz point problem in magnetic systems [1]. The Lifshitz point is also of interest for phase transitions in liquid crystals and other systems, see [2, 3] and references therein. The case of two component spins has been analyzed in detail by Grinstein, Pelcovits, Nelson and Toner [3–6].

When the spin has three components we shall see that there exist pointlike topological defects that might strongly influence the thermodynamic behaviour of the system. This is the prime motivation for studying (1.1). The action of the defects is logarithmically divergent. The aim of this paper is to prove that there exist such defects and to estimate their interaction energy.

One could add a term proportional to $(\nabla \mathbf{S})^4$ to the functional (1.1), where ∇ is the gradient in the xy-plane, without substantially changing our results. We choose to study (1.1) for the sake of simplicity. However, a term proportional to $(\nabla \mathbf{S})^2$ would cause a linear divergence in the action of the defects, see [7].