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# Cylindrically and Spherically Symmetric Monopoles in SU(3) Gauge Theory

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**Abstract.** We apply to the Atiyah-Ward ansätze a systematic procedure locating symmetric monopoles in SU(3) gauge theory broken to U(1) × U(1). In particular we recover the known spherically symmetric monopole as a limit of a cylindrically symmetric separated two monopole solution in SU(3). We also discuss the spherically symmetric monopole in SU($n$). This latter is the only instance where we have properly shown the smoothness of the Higgs and gauge fields.

## Introduction

Over the past year there has been a great deal of progress in the understanding of monopoles in gauge theories. It commenced with the discovery that the Atiyah-Ward construction [1] of self dual solutions is better suited to monopoles than to the instantons which motivated it. The doubly charged SU(2) monopole found by Ward [2] and independently by Forgacs et al. [3] was generalised by Prasad and Rossi and Forgacs et al. [4] to higher charges. As yet all these monopoles were located at a single point and had cylindrical symmetry. Ward [5] then produced the first true multimonopole, two charge one monopoles separated by a small distance. Corrigan and Goddard [6] generalised this to a $4n-1$ parameter family of SU(2) multimonopoles.

SU(3) is clearly the next place to look and, again, Ward [7] has found a one parameter family of cylindrically symmetric monopoles which have as spherically symmetric limit the solution for SU(3) broken to U(2) familiar from earlier work.

In this paper we shall look for such families when SU(3) is broken to U(1) × U(1).

Monopoles are finite energy solutions of the Bogomolny equation

$$D_i\phi = \pm\tfrac{1}{2}\varepsilon_{ijk}F_{jk} \qquad i,j,k=1,2,3\,, \tag{1.1}$$

where $\phi$ is the Higgs field, in the adjoint representation of SU($n$), $D_j$ is the covariant derivative $(\partial_i + iA_j)$ and $F_{jk}$ the space part of the field strength tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]\,. \qquad A_\mu = A_\mu^\dagger\,.$$