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Exponential Bounds and Absence of Positive Eigenvalues for N-Body Schrödinger Operators

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Abstract. For a large class of N-body potentials V we prove that if ψ is an eigenfunction of $-\Delta+V$ with eigenvalue E then $\sup\{\alpha^2+E:\alpha\geqq 0, \exp(\alpha|x|)\psi\in L^2\}$ is either a threshold or $+\infty$. Consequences of this result are the absence of positive eigenvalues and "optimal" L^2-exponential lower bounds.

I. Introduction

In this paper we will be concerned with the N-body Schrödinger operator

$$H=H_0+V, \tag{1.1}$$

$$V=\sum_{1\leqq i<j\leqq N} V_{ij}, \tag{1.2}$$

in $L^2(\mathbb{R}^{\nu(N-1)})$. Here H_0 arises from the operator

$$\tilde{H}_0=-\sum_{i=1}^{N}\Delta_i/2m_i \tag{1.3}$$

by removing the center of mass (see [16, 17] and Sect. II for more details). Each V_{ij} is multiplication by a real-valued function $v_{ij}(x_i-x_j)$, where here $x\in\mathbb{R}^{\nu N}$ is written $x=(x_1,\ldots,x_N)$. Let h_0 be $-\Delta$ in $L^2(\mathbb{R}^\nu)$. We assume in what follows that each two-body potential v_{ij} satisfies

(a) $$v_{ij}(1+h_0)^{-1} \quad \text{is compact}, \tag{1.4}$$

(b) $$(1+h_0)^{-1}(y\cdot\nabla v_{ij})(1+h_0)^{-1} \quad \text{is compact}. \tag{1.5}$$

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