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The Unpredictability of Quantum Gravity

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Abstract. Quantum gravity seems to introduce a new level of unpredictability into physics over and above that normally associated with the uncertainty principle. This is because the metric of spacetime can fluctuate from being globally hyperbolic. In other words, the evolution is not completely determined by Cauchy data at past or future infinity. I present a number of axioms that the asymptotic Green functions should obey in any reasonable theory of quantum gravity. These axioms are the same as for ordinary quantum field theory in flat spacetime, except that one axiom, that of asymptotic completeness, is omitted. This allows pure quantum states to decay into mixed states. Calculations with simple models of topologically non-trivial spacetime indicate that such loss of quantum coherence will occur but that the effect will be very small except for fundamental scalar particles, if any such exist.

1. Introduction

In the eighteenth and nineteenth centuries it was believed that physics was completely deterministic. That is to say, the classical physical laws determined the exact values of the coordinates and the conjugate momenta of a physical system from their values at one time. In practice, the calculation usually became too complicated for systems of more than a few degrees of freedom, so people resorted to statistical treatments in which they did not attempt to predict the exact state of the system, but only certain gross properties such as the pressure or the temperature. The use of classical statistical mechanics was regarded only as a matter of convenience, however, in principle it was believed that one could predict the exact state of the system.

With the advent of quantum mechanics in the 1920's, it was realized that one could predict exactly *either* the values of the co-ordinates *or* the values of the momenta, but not both. More precisely, the most that one could predict exactly were the values of a complete commuting set of observables and that only if the system happened to be in an eigenstate of that set of observables. Thus, roughly speaking, one's ability to make precise predictions was cut in half. As in the case of

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