Commun. Math. Phys. 87, 353–364 (1982)

Communications in Mathematical Physics

Derivations Commuting with Abelian Gauge Actions on Lattice Systems

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Abstract. Let τ be an action of a compact abelian group G on a C^*-algebra $\mathscr{A}$, and assume that the fixed-point subalgebra $\mathscr{A}^\tau$ is an AF-algebra. We show that if δ is a closed *-derivation on $\mathscr{A}$ commuting with τ, and the restriction of δ to $\mathscr{A}^\tau$ generates a one-parameter group of *-automorphisms, then δ itself is a generator. In particular, the result applies if τ is an infinite product action of G on a UHF algebra. Furthermore, if in this situation δ_1 and δ_2 are two derivations both satisfying the hypotheses on δ, and δ_1 and δ_2 have the same restriction to $\mathscr{A}^\tau$, then there exists a one-parameter subgroup of the action τ with generator δ_0 such that $D(\delta_1)\cap D(\delta_2)\cap D(\delta_0)$ is a joint core for the three derivations, and $\delta_2 = \delta_1 + \delta_0$ on this core.

1. Introduction

Let δ be a closed *-derivation with dense domain $D(\delta)$ in a C^*-algebra $\mathscr{A}$. Assume that δ commutes with a strongly continuous action τ of a compact abelian group G as *-automorphisms on $\mathscr{A}$. It was shown in [4] that if δ vanishes identically on the fixed-point algebra $\mathscr{A}^\tau = \{A\in\mathscr{A} : \tau(g)(A) = A,\ g\in G\}$, then δ is the infinitesimal generator of a strongly continuous one-parameter group of *-automorphisms of $\mathscr{A}$. Briefly, we say that δ is a generator. By a simple perturbation argument, it follows that the assumption $\delta|\mathscr{A}^\tau = 0$ may be weakened to the condition that $\delta|\mathscr{A}^\tau$ is inner. An example in [4] showed that it is not enough to assume that $\delta|\mathscr{A}^\tau$ is a generator on $\mathscr{A}^\tau$. In this example, $\mathscr{A}$ is abelian, and there is a geometric obstruction preventing δ from being a generator: Along the integral curves of the propagator, points burst into fibres, and conversely, fibres merge into points, in a finite time, [1].

On the other hand, Kishimoto and Robinson [15] showed that if one adds the assumption that $\mathscr{A}$ has an identity, and $\mathscr{A}^\tau(\gamma)^*\mathscr{A}^\tau(\gamma) = \mathscr{A}^\tau$ for all $\gamma\in\hat{G}$, where $\mathscr{A}^\tau(\gamma) =$

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0010-3616/82/0087/0353/$02.40