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Resonances for the AC-Stark Effect*

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Abstract. The resonance problem for the AC-Stark effect is discussed. We prove that all bound states of the system $-(1/2)\Delta + V(x)$ will turn into resonances after an AC-electric field is switched on and the order of the imaginary part of a resonance is determined by the number of the photons it takes to ionize the bound state which is turning into the resonance; if two bound states have energy difference of the photon, there exists a state which oscillates between the two states for a long time.

1. Introduction

Suppose that a quantum particle of mass m and charge e in a potential field $V(x)$ is subject to an alternating electric field $\mu E \cos \omega t$. Then the Schrödinger equation for the motion of the particle is written as

$$i\hbar\, \partial u/\partial t = [-(\hbar^2/2m)\Delta + V(x) - \mu e Ex \cos \omega t]\, u. \tag{1.1}$$

Here $\mu > 0$ is the strength of the field, $E \in \mathbb{R}^3, |E| = 1$ is the direction, ω is the frequency, $\hbar = h/2\pi$ and h is Planck's constant;

$$-\Delta = -(\partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2).$$

The purpose of this paper is to study the resonances for Eq. (1.1). We shall argue that the resonances should be defined as the poles of the resolvent of the "coupled photon-particle Hamiltonian" $-i\hbar\, \partial/\partial t - (\hbar^2/2m)\Delta + V(x - \mu e m^{-1} \omega^{-2} E \cos \omega t)$ in the second Riemann sheet and show, in particular, the following two results under suitable conditions on $V(x)$:

(A) For sufficiently small μ and almost all ω, all the bound states $\{(\phi_j(x), -k_j^2)\}$ of $H = -(\hbar^2/2m)\Delta + V(x)$ will turn into the resonances $\{(\phi_j(t,x,\mu), \lambda_j)\}$ and the imaginary part of λ_j is determined as $\operatorname{Im} \lambda_j = C_j(\omega)\mu^{2n} + O(\mu^{2n+1})$, $C_j(\omega) < 0$, where n is the smallest integer such that $-k_j^2 + n\hbar\omega > 0 : (\phi_j(t,x,\mu), \phi_j(x)) = e^{-i\lambda_j t/\hbar} + O(\mu)$ uniformly in $t \geqq 0$ (Theorem 3.5 and 3.6).

(B) If two bound states $(\phi_j(x), -k_j^2)$ and $(\phi_i(x), -k_i^2)$ of H have the energy

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