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# Some Jacobi Matrices with Decaying Potential and Dense Point Spectrum

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**Abstract.** We discuss doubly infinite matrices of the form $M_{ij}=\delta_{i,j+1}+\delta_{i,j-1}+V_i\delta_{ij}$ as operators on $\ell^2$. We present for each $\varepsilon>0$, examples of potentials $V_n$ with $|V_n|=O(n^{-1/2+\varepsilon})$ and where $M$ has only point spectrum. Our discussion should be viewed as a remark on the recent work of Delyon, Kunz, and Souillard.

## 1. Introduction

During the past few years, numerous results have appeared showing that most of the spectrum of Schrödinger operators $-\Delta+V$ is purely absolutely continuous (a.c.). This includes not only the "short range case", $V(x)=O(x^{-1-\varepsilon})$ at infinity (see e.g. [1]), but also the long range case where $V(x)=O(x^{-\varepsilon})$ so long as derivatives decay (see e.g. [5]) and the highly oscillatory case where $V(x)=O(x^{-\alpha}\sin x^{\beta})$ for suitable $\alpha$, $\beta$ (see e.g. [8]). While there are examples of Pearson [7] with decaying $V$ where the spectrum is not a.c., the rate is so slow that one might be led to suspect that reasonable (say power) decay always leads to a.c. spectrum. (The other recently constructed examples with non-a.c. behavior, namely random [4] and special almost periodic potentials [2], of course, have no decay at infinity.) Our goal here is to indicate that there are power decaying potentials which lead to non-a.c. spectrum.

In the end our examples will be for $-\Delta$ replaced by a finite difference operator; explicitly, on $\ell^2(Z)$ let

$$(M_0u)(i)=u(i+1)+u(i-1),$$

and

$$(Vu)(i)=V(i)u(i).$$

We will consider $M_0+V$ on $\ell^2(Z)$. The potential $V$ will obey $V(n)\sim O(n^{-1/2+\varepsilon})$, but differences $V(n+1)-V(n)$ will also be $O(n^{-1/2+\varepsilon})$. The non-random analog will be potentials of the form $x^{-\alpha}\sin x$ with $\alpha<\frac{1}{2}$. It is an interesting fact that the known

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