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Generalized Transition Probability, Mobility and Symmetries

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Abstract. In the framework of Mackey's description of a physical system, the *generalized transition probability*, as defined in an earlier paper, is shown to be non-decreasing while the system evolves, and invariant when the evolution is reversible. It is also invariant under a natural action of the space-time symmetry group.

1. Introduction

As in [1], we shall adopt Mackey's description of a physical system in terms of a set $\mathscr{S}$ of *states*, a set $\mathscr{O}$ of *observables*, and a structure function $\not{p}$ representing the probability distributions associated with the measurements of the observables on the states [2]. This broad framework is enough to define a *generalized transition probability* $T(\alpha,\beta)$ on $\mathscr{S} \times \mathscr{S}$ (or, equivalently, a distance function $\mathscr{d}(\alpha,\beta)$ which turns $\mathscr{S}$ into a metric space). [1, 3–5].

Additional structure must be specified in order to exhibit how the generalized transition probability is related to the dynamical evolution of the system, and to the space-time symmetry group in a relativistic theory.

Following Mielnik [6, 7], we shall represent the set of all possible evolutions of the physical system by a *mobility semigroup* $\mathscr{M}$. Its natural action on $\mathscr{S}$ and $\mathscr{O}$ will turn out to be such that the generalized transition probability between any pair of evolving states cannot decrease with time. Under an additional reversibility assumption, $\mathscr{M}$ becomes a group and T is preserved. Similarly, in a relativistic theory, the space-time symmetry group $\mathscr{G}$ must have a natural action on $\mathscr{S}$ and $\mathscr{O}$ such that the function T be preserved.

We shall conclude by remarking that, in this perspective, the purely *metric* aspects of the quantum-mechanical formalism, directly related to observation *via* the transition probability, acquire a *primary* significance, while the underlying linear structure, and all its important consequences, appear in a certain sense as *derived* elements—a remark which seems pertinent both to the justification of the established formalism and to the search for its possible extensions or modifications.

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