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Multi-Instantons Localized at the Origin

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Abstract. We obtain a family of self-dual Yang-Mills fields in an SU(2) gauge theory. Some of them describe pseudoparticles with arbitrary topological numbers and with action densities concentrated around the origin.

1. Introduction

By now, much is known about self-duality equations (SDE) for SU(2) gauge theories. In principle, Atiyah et al. [1] have solved the problem completely, but only a restricted number of solutions are explicitly known and understood as solitons by knowledge of their action or energy density. The most popular one is the k-instanton discovered by 'tHooft [2]. It corresponds essentially to a superposition of k widely separated instantons [3].

Recently, research for multi-monopoles has led to new (time independent) solutions [4]; these have a cylindric symmetry, finite energy and their energy density is maximal on a circle in such a way that most of the energy is concentrated in a torus-like region of space.

In this paper, we exhibit a class of time dependent solutions with finite action and an action density maximal on a circle in Euclidean space-time. To obtain such solutions, we require a particular transformation law of the fields under the subgroup SO(2)×SO(2) of rotations in the x_1, x_2 plane and in the x_0, x_3 plane. The solutions belong to a large class obtained in [8] with different motivations. In Sect. 2 of the paper, we rapidly explain the ansatz and the construction; then we study in Sect. 3 the physically relevant solutions. Some conclusions are drawn in Sect. 4.

Let us first write the equations to be satisfied; in order to study the SDE

$$F_{\mu\nu} = \tilde{F}_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu\varrho\sigma} F^{\varrho\sigma} \tag{1.1}$$

for the gauge field $F_{\mu\nu}$ defined as usual in terms of a gauge potential A_μ by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$, we have used the Yang formalism. It works with the light-like coordinates

$$Y = \frac{1}{\sqrt{2}}(x^0 - ix^3), \qquad Z = \frac{1}{\sqrt{2}}(x^2 + ix^1), \qquad \bar{Y} = Y^*, \qquad \bar{Z} = Z^* \tag{1.2}$$

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