Analyticity Properties of the Weakly Coupled Double-Well Model

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Abstract. In this note we study lattice Φ^4-models with Hamiltonian

$$H = \tfrac{1}{2}(\varphi, -\Delta\varphi) + \lambda\Sigma\left(\varphi_i^2 - \frac{m^2}{8\lambda}\right)^2$$

and Gaussian boundary conditions. Using the polymer expansion we obtain analyticity of the pressure and the correlation functions in the infinite volume limit in a region

$$\left\{\lambda \mid |\lambda| < \varepsilon, |\arg\lambda| < \frac{\pi}{2} - \delta\right\}$$

for every $\delta > 0$.

1. Introduction

Let us consider the $P(\varphi)$-theory on the ν-dimensional lattice $\mathbb{Z}^\nu$ with partition function in the finite volume Λ given by

$$Z_\Lambda(\lambda, m) =$$

$$\int_{\mathbb{R}^{|\bar{\Lambda}|}} \exp\left(-\tfrac{1}{2}(\varphi, -\Delta_\Lambda\varphi) - \sum_{i\in\Lambda} P(\varphi_i) - \frac{m^2}{2}\sum_{i\in\partial_+\Lambda}\left(\varphi_i - \frac{m}{\sqrt{8\lambda}}\right)^2\right)\prod_{i\in\bar{\Lambda}}d\varphi_i, \quad (1.1)$$

where

$$\partial_+\Lambda = \{i\in\mathbb{Z}^\nu\backslash\Lambda \mid \exists j\in\Lambda, \quad |i-j| = 1\}$$

and

$$\bar{\Lambda} = \Lambda \cup \partial_+\Lambda.$$

We study the case

$$P(\varphi) = \lambda\left(\varphi^2 - \frac{m^2}{8\lambda}\right)^2, \qquad (1.2)$$