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On the Convergence of Planar Diagram Expansions

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Abstract. Renormalizable quantum field theories whose perturbation expansions are described by planar Feynman diagrams only, such as SU(∞) gauge theory, are considered in 4 dimensional Euclidean space. For studying asymptotic properties of the perturbation series one might wish to isolate first all those planar diagrams that do not contain any ultraviolet divergent subgraphs. In this paper it is proved that this infinite set of diagrams, when summed, converges within a finite radius of convergence for the coupling constant.

1. Introduction

In spite of considerable efforts is is still not known how to compute physical quantities accurately in any four dimensional quantum field theory with strong interactions. It seems quite likely that if any strong interaction field theory exists in which accurate computations can be done, then that must be an asymptotically free, non-Abelian gauge theory. In such theories the small-distance structure is completely described by the solutions of the renormalization group equations; and there are reasons to believe that the continuum theory can be uniquely defined as a limit of a lattice gauge theory, when the size of the meshes of the lattice tends to zero, together with the coupling constant, in a way prescribed by this renormalization group. Indeed one can prove using the formalism of [1] that this limit exists up to any finite order in the perturbation expansion for small coupling.

However, this result has not been extended beyond perturbation expansion. Therefore, the popular belief that theories such as "quantum chromodynamics" are mathematically well-founded might be preposterous. In [2] this author speculated that physical changes in the QCD theory might be necessary in order to tame its (suspected) disease: perhaps the number of colors N has to tend to infinity at increasing energies. Arguments were presented that hold some promise that a mathematically accurate continuum field theory can be constructed along these lines, but much more work on the expansions in the effective coupling constant $\tilde{g}^2 = g^2 N$ must be done.

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