Commun. Math. Phys. 86, 337–362 (1982)

Communications in
Mathematical
Physics

Perturbation about the Mean Field Critical Point

Jean Bricmont[†][⋆][1] and Jean-Raymond Fontaine[°][⋆][2]

[1] Department of Mathematics, Princeton University, Princeton, NJ 08544, USA
[2] Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, USA

Appendix

Eugene Speer

Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, USA

Abstract. We consider two models that are small perturbations of Gaussian or mean field models: the first one is a double well $\lambda/4\phi^4 - \sigma/2\phi^2$ perturbation of a massless Gaussian lattice field in the weak coupling limit ($\lambda \to 0$, σ proportional to λ). The other consists of a spin 1/2 Ising model with long-range Kac type interactions; the inverse range of the interaction, γ, is the small parameter. The second model is related to the first one via a sine-Gordon transformation. The lattice $\mathbb{Z}^d$ has dimension $d \geqq 3$.

In both cases we derive an asymptotic estimate to first order (in λ or γ^2) on the location of the critical point. Moreover, we prove bounds on the remainder of an expansion in λ or γ around the Gaussian or mean field critical points.

The appendix, due to E. Speer, contains an extension of Weinberg's theorem on the divergence of Feynman graphs which is used in the proofs.

Introduction

It is well known that rigorous and accurate estimates on the location of the critical point of statistical mechanical systems are in general very difficult to obtain. This is related to the fact that high and low temperature expansions are not known to converge up to the critical temperature. However, for models that are close to Gaussian or mean-field theories, we can obtain more detailed information.

Specifically, we shall consider three cases. The nearest-neighbour Ising model on $\mathbb{Z}^d$ in the limit of $d \to \infty$; a lattice field theory in the weak coupling limit, and an Ising model with weak long range interaction of Kac type. For the nearest neighbour Ising model it is straightforward to show, combining the infrared bounds [1] and Fisher's mean field bounds on T_c [2] that

$$T_c(d)/2d = 1 - 1/2d + o(1/d)$$

† Supported by NSF Grant # MCS 78-01885
° Supported by NSF Grant # PHY 78-15920
⋆ Present address: Institut de Physique Theorique, Université de Louvain, Belgium

0010-3616/82/0086/0337/$05.20