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All Massless, Scalar Fields with Trivial S-Matrix are Wick-Polynomials

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Abstract. We extend a result about non-interacting fields given by Buchholz and Fredenhagen. Consider a massless, scalar field ϕ in $3+1$ dimensional space-time which does not interact. The corresponding Hilbert space is assumed to be the Fockspace H of the free massless field A. This implies – as we show in the first part – that all n-point-functions are rational functions of their arguments. In the second part we use this fact to construct a symmetric, traceless tensorfield $\phi^{\mu_1 \cdots \mu_n}$, relatively local to the original field ϕ, and connecting the vacuum with the one particle states. In the last part we prove $\phi^{\mu_1 \cdots \mu_n}$ to be relatively local to the free field A.

0. Introduction

In a series of papers Buchholz establishes a frame for a scattering theory for massless particles in $3+1$ dimensional space-time [1]:

Let $A(x)$ be the free, massless, scalar field acting in the Fockspace H. Let $\phi(x)$ be a real, scalar field which transforms under the same unitary representation of the Poincaré group as $A(x)$. The corresponding Hilbert space is assumed to be the Fockspace H. We identify $A(x)$ with the incoming field $\phi^{\text{in}}(x)$, respectively the outgoing field $\phi^{\text{out}}(x)$. In [1] Buchholz shows that

$$[\phi^{\text{in}}(x), \phi(y)] = 0 \quad \text{for} \quad y - x \in V^- \text{ (backward cone)}$$

and

$$[\phi^{\text{out}}(x), \phi(y)] = 0 \quad \text{for} \quad y - x \in V^+ \text{ (forward cone)}.$$

We want to prove the following:

Theorem. *If $\phi(x)$ has a trivial S-matrix, then $\phi(x)$ is relatively local to the free field $A(x)$.*

This theorem extends the result given by Buchholz and Fredenhagen [2]. In their paper they show first that ϕ can be decomposed into a finite sum of fields ϕ_d with

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