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General Lower Bounds for Resonances in One Dimension*

Evans M. Harrell II

Department of Mathematics, The Johns Hopkins University, Baltimore, MD 21218, USA

Abstract. Lower bounds are derived for the magnitude of the imaginary parts of the resonance eigenvalues of a Schrödinger operator

$$-\frac{d^2}{dx^2}+V(x)$$

on the line, depending only on the support and bounds of V and on the real part of the resonance eigenvalue. For example, if the resonance eigenvalue is denoted $E+i\varepsilon$, then there exist C and ℓ_0 depending only on $\|V\|_\infty$ and E such that if the support of V is contained in an interval of length $\ell>\ell_0$, then

$$|\varepsilon|>\frac{m^3\sqrt{E}}{(m+\sqrt{E})^2}\exp(-m\ell)(1-C\ell^{-1}),$$

where $m=\|V(x)-E\|_\infty^{1/2}$.

Spencer has recently raised the question of whether there is a lower bound for the magnitude of the imaginary parts of the resonance eigenvalues for a quantum-mechanical particle in a compactly supported potential that is randomly generated, and therefore neither necessarily regular nor known at all in detail [1]. The purpose of this note is to derive such lower bounds in the one-dimensional case assuming knowledge of the real part. The many somewhat distinct notions of resonance for operators $-\Delta+V(x)$ on $L^2(\mathbb{R}^n)$ become all more or less the same if $n=1$ and V is compactly supported. A physically motivated definition would be a complex value of k for which a scattering state for

$$\frac{-d^2}{dx^2}+V(x)$$

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