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On the Atiyah-Drinfeld-Hitchin-Manin Construction for Self-Dual Gauge Fields

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Abstract. The vector spaces A, B, C, in terms of which the general construction due to Atiyah, Drinfeld, Hitchin and Manin for self-dual gauge fields defined over some region of Euclidean space is phrased, are shown to be expressible in terms of the spaces spanned by the solutions of certain linear covariant differential equations depending on the gauge field. The corresponding linear maps between A and B, B and C are given with the properties required by ADHM and the results then necessary to verify the construction informally proved. The local problems associated with assuming the gauge field to obey the self-duality equations are separated from the global problems of assuring the required boundary conditions for a particular solution. With suitable global conditions C is shown to be the dual of A and a natural scalar product defined on B so as to reconstruct the gauge field in the standard form given by the construction. A discussion is given of the requirements entailed by the condition of a symmetry on the gauge field and the relation to the usual cohomological treatment is outlined in an appendix.

1. Introduction

Given that non-Abelian gauge theories are an essential feature in our theoretical description of particle physics it seems desirable to explore their mathematical structure in some detail. In this context the analysis of possible solutions of the classical equations obeyed by the gauge field is of interest. An important set, although of course in no way is it the general case, are the solutions of the Euclidean self-duality equations. For a gauge field $A_\mu = A_\mu^a t_a$, $\{t_a\}$ being a set of matrix generators forming a basis for the Lie algebra of a gauge group $\mathscr{G}$, these read

$$F_{\mu\nu} = {}^*F_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]\,. \tag{1.1}$$

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