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Shock Profile Solutions of the Boltzmann Equation*

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Abstract. Shock waves in gas dynamics can be described by the Euler Navier–Stokes, or Boltzmann equations. We prove the existence of shock profile solutions of the Boltzmann equation for shocks which are weak. The shock is written as a truncated expansion in powers of the shock strength, the first two terms of which come exactly from the Taylor $\tanh(x)$ profile for the Navier–Stokes solution. The full solution is found by a projection method like the Lyapunov–Schmidt method as a bifurcation from the constant state in which the bifurcation parameter is the difference between the speed of sound c_0 and the shock speed s.

1. Introduction

Shock waves are one of the most important features of gas dynamics. They can be understood from several different theories, and for steady plane shock waves the different descriptions have been well developed mathematically. By the Euler equations, and the resulting Rankine–Hugoniot conditions, a shock is described as a jump discontinuity in density, velocity, and temperature from $(\rho_-, \mathbf{u}_-, T_-)$ on the left to $(\rho_+, \mathbf{u}_+, T_+)$ on the right, which translates steadily at speed s[4]. If viscosity and heat conduction are included through the compressible Navier–Stokes equations, the shock wave is found to be a smooth profile which translates uniformly at speed s and smoothly interpolates between the asymptotic values $(\rho_-, \mathbf{u}_-, T_-)$ at $x = -\infty$ and $(\rho_+, \mathbf{u}_+, T_+)$ at $x = +\infty$ [9, 21].

For a weak shock this provides shock profiles very close to those observed experimentally. But for strong shock waves more realistic results are obtained from the Boltzmann equation of kinetic theory, which includes a statistical description of the molecular interactions within the gas. The Boltzmann shock profile translates uniformly at speed s and interpolates between two velocity distribution functions $F_-(\xi)$ at $x = -\infty$ and $F_+(\xi)$ at $x = +\infty$ which are uniform

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