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Energy Dependence of the Scattering Operator II

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Abstract. We study the energy dependence of the scattering operator for a two-body model of electron scattering from a neutral molecule. We show that the methods of the first paper can be applied even though the dipole moment of the molecule is non-zero, and prove continuity of the scattering operator $S(E)$ as E varies, in a very strong sense.

1. Introduction

We study the elastic scattering of an electron from a neutral molecule in the two-body approximation. That is, we study the scattering between $H = -\Delta$ and $K = H + V$ on $\mathscr{H} = L^2(\mathbb{R}^3)$, where

$$V(x) = \int_{\mathbb{R}^3} \frac{1}{|x-y|} \mu(dy).$$

We assume that the charge distribution μ is a signed measure of finite total mass and zero net charge, that is

$$\int \mu(dy) = 0.$$

We also assume that the dipole moment a of μ, given by

$$a = \int y \mu(dy)$$

is non-zero, so that the potential V has the asymptotic form

$$V(x) = \frac{a \cdot x}{r^3} + O(r^{-3})$$

at infinity, where $r = |x|$. To ensure that a is finite we assume μ has support within $\{x : |x| \leqq R\}$ for some $R < \infty$. Our methods could, however, easily cope with a charge distribution with exponential tails at infinity.

The potential V is fairly well-behaved, and there are a variety of techniques [1, 7] which ensure that the wave operators between H and K exist and are complete. We are interested in studying the energy dependence of the scattering operator

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