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Normal Product States for Fermions and Twisted Duality for CCR- and CAR-Type Algebras with Application to the Yukawa$_2$ Quantum Field Model

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Abstract. We present sufficient conditions that imply duality for the algebras of local observables in all Abelian sectors of all locally normal, irreducible representations of a field algebra if twisted duality obtains in one of these representations. It is verified that the Yukawa$_2$ model satisfies these conditions, yielding the first proof of duality for the observable algebra in all coherent charge sectors in this model. This paper also constitutes the first verification of the assumptions of the axiomatic study of the structure of superselection sectors by Doplicher, Haag and Roberts in an interacting model with nontrivial sectors. The existence of normal product states for the free Fermi field algebra and, thus, the verification of the "funnel property" for the associated net of local algebras are demonstrated.

1. Introduction

In the algebraic approach to relativistic quantum field theory (for an introduction and motivation, see [24]), an intriguing property called duality became a natural object of study. For algebraic quantum field theory the basic structure is a net of $*$-algebras $\{\mathfrak{A}(\mathcal{O})\}$ (generally taken to be von Neumann algebras, which we henceforth assume), wherein to each bounded space-time region $\mathcal{O}$ is associated an algebra $\mathfrak{A}(\mathcal{O})$. This net of algebras is required to satisfy certain properties that are physically motivated when one assumes that the self-adjoint elements of $\mathfrak{A}(\mathcal{O})$ represent measurements performed in $\mathcal{O}$ (observables in $\mathcal{O}$). A crucial property is that of locality, which is an expression in this framework of the requirement that measurements in spacelike separated regions can not influence each other. Locality is represented mathematically by demanding that observables A_1 and A_2 localized in such spacelike separated regions $\mathcal{O}_1$ and $\mathcal{O}_2$ must commute, i.e. $\mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2')'$, where primes on algebras signify the commutant of the algebra and primes on space-time regions signify the spacelike complement of the region. Duality strengthens this requirement by demanding that any observable that commutes with all observables

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