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Borel Summability of the $1/N$ Expansion for the N-Vector [$O(N)$ Non-Linear σ] Models

J. Fröhlich[1]*, A. Mardin[2], and V. Rivasseau[3]**

1 Institut des Hautes Etudes Scientifiques, 35, Route de Chartres, F-91440 Bures-sur-Yvette, France
2 C.P.T., Ecole Polytechnique, F-91128 Palaiseau Cédex, France
3 The Institute for Advanced Study, Princeton, NJ 08540, USA

Abstract. We construct an analytic interpolation in $1/N$ for the N-vector [$O(N)$ non-linear σ] models with N-component fields on a lattice. This interpolation, valid at sufficiently high temperatures, extends over a large domain in the complex plane containing the half plane $\mathrm{Re}\,(1/N) > 0$. We use this result to show that the $1/N$ expansion of the free energy density and of the correlation functions is Borel summable in the thermodynamic limit and at high temperature.

1. Introduction, Notations and Main Results

In this paper we continue a mathematically rigorous analysis of the $1/N$ expansion in the N-vector models, initiated by A. Kupiainen [1, 2]. Kupiainen has shown that the $1/N$ expansion is asymptotic for two families of models, the N-vector models on a simple, (hyper) cubic lattice $\mathbb{Z}^d$, $d = 2, 3, 4, \ldots$, at temperatures above the critical temperature of the spherical model ($N = \infty$), and a class of weakly coupled N-component $\lambda|\phi|^4$ models in two space-time dimensions. A careful analysis of the $1/N$ expansion for the three-dimensional $O(N)$ σ-models in the continuum limit has been carried out by I. Aref'eva [3] who, however, has not determined its nature. For a summary of the history of $1/N$ expansions and references to important, earlier work, see Kupiainen's papers [1, 2].

A natural problem is to study the analyticity properties in $1/N$ and to determine the summability properties of the $1/N$ expansion for the models mentioned above. Billionnet and Renouard have recently proven that the $1/N$ expansion for weakly coupled N-component $\lambda|\phi|^4$ models in two dimensions is Borel-summable [4]. In this paper we establish the same result for the $O(N)$ non-linear σ-models on a lattice of arbitrary dimension, at high temperature. The methods used in this paper are different from the ones in [4]. In [4] the main technical difficulty appears in the construction of the continuum (ultraviolet) limit. Here we do not construct

* Address after August 1982: Theoretical Physics, ETH, CH-8091 Zürich, Switzerland
** On leave from C.P.T., Ecole Polytechnique, F-91128 Palaiseau Cédex, France

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